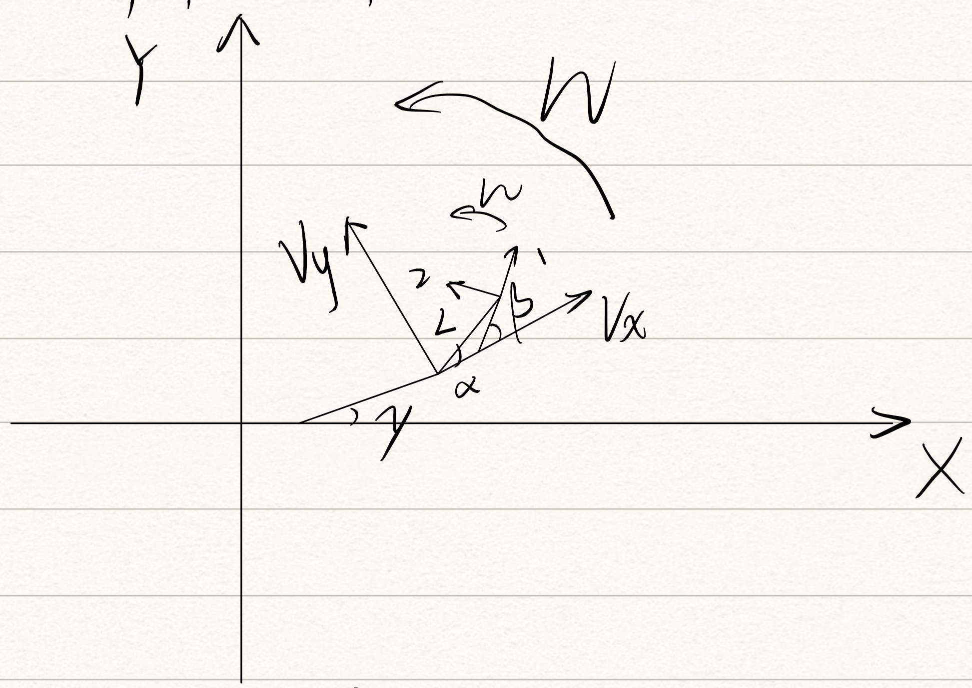
圆弧解算及反算标定 L, α



1, 2 为定位坐标系，V_x, V_y 为小车坐标系，

X, Y 为世界坐标系，

码盘 → 小车

有

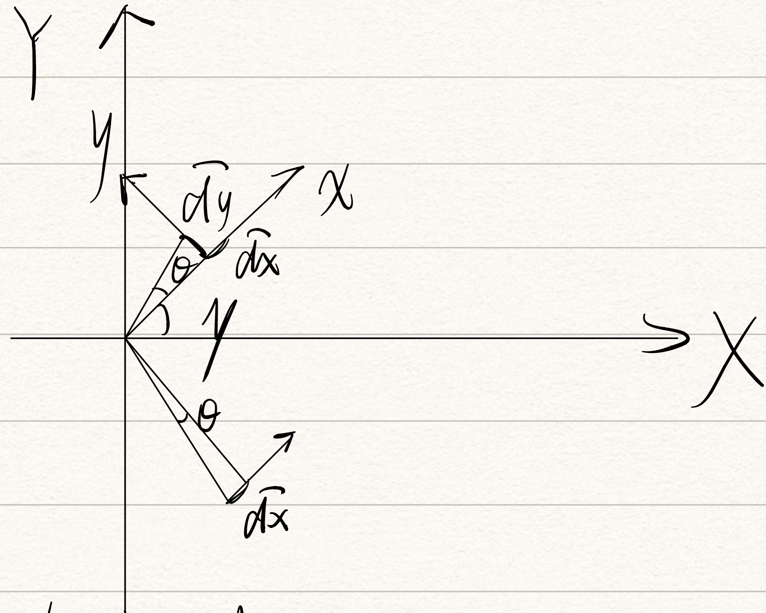
$$\underbrace{\begin{pmatrix} V_x \\ V_y \\ w \end{pmatrix}}_{Q_1} = \underbrace{\begin{pmatrix} \cos\beta & \sin\beta & L\cdot\sin\alpha \\ -\sin\beta & \cos\beta & -L\cdot\cos\alpha \\ 0 & 0 & 1 \end{pmatrix}}_{A} \underbrace{\begin{pmatrix} V_1 \\ V_2 \\ w \end{pmatrix}}_{Q_2}$$

在 samp-time 内

$$\begin{pmatrix} \Delta x \\ \Delta y \\ \Delta\theta \end{pmatrix} = \begin{pmatrix} \cos\beta & \sin\beta & L\cdot\sin\alpha \\ -\sin\beta & \cos\beta & -L\cdot\cos\alpha \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \Delta 1 \\ \Delta 2 \\ \Delta\theta \end{pmatrix}$$

以 $\bar{dx}, \bar{dy}, \theta$ 代表 $\Delta x, \Delta y, \Delta\theta$，

以下为圆弧更新模型



以 dy 为例

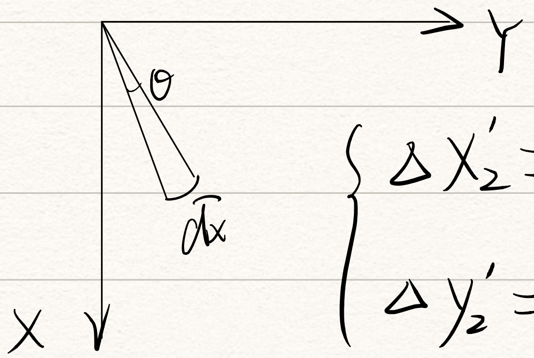
$$\Delta X = \frac{\bar{dy}}{\theta}\cdot\left(\cos(\gamma+\theta)-\cos\gamma\right)$$

对于曲率半径

$$K = \frac{\bar{dy}}{\theta} = \frac{\frac{1}{2}\bar{dy}}{\sin\frac{\theta}{2}} = \frac{\bar{dy}}{2\sin\frac{\theta}{2}}$$

故
$$\begin{cases} \Delta X_1 = \dfrac{\bar{dy}}{2\sin\frac{\theta}{2}}\cdot\left(\cos(\gamma+\theta)-\cos\gamma\right) \\ \Delta Y_1 = \dfrac{\bar{dy}}{2\sin\frac{\theta}{2}}\cdot\left(\sin(\gamma+\theta)-\sin\gamma\right) \end{cases}$$

将坐标系顺时针旋转 $\frac{\pi}{2}$ 后计算 dx 的影响



$$\begin{cases} \Delta X_2' = \dfrac{\bar{dx}}{2\sin\frac{\theta}{2}}\cdot\left(\cos(\gamma+\theta)-\cos\gamma\right) = -\Delta Y_2 \\ \Delta Y_2' = \dfrac{\bar{dx}}{2\sin\frac{\theta}{2}}\cdot\left(\sin(\gamma+\theta)-\sin\gamma\right) = \Delta X_2 \end{cases}$$

故
$$\begin{cases} \Delta X_2 = \dfrac{\bar{dx}}{2\sin\frac{\theta}{2}} \cdot \left(\sin(\gamma+\theta) - \sin\gamma \right) \\ \Delta Y_2 = -\dfrac{\bar{dx}}{2\sin\frac{\theta}{2}} \left(\cos(\gamma+\theta) - \cos\gamma \right) \end{cases}$$

由
$$\begin{cases} \Delta X = \Delta X_1 + \Delta X_2 \\ \Delta Y = \Delta Y_1 + \Delta Y_2 \end{cases}$$

且代入
$$\begin{cases} dx = \cos\beta\,\Delta 1 + \sin\beta\,\Delta 2 + L\theta \cdot \sin\alpha \\ dy = -\sin\beta\,\Delta 1 + \cos\beta\,\Delta 2 - L\theta\cos\alpha \end{cases}$$

得:

$$\begin{cases} \Delta X = \dfrac{1}{2\sin\frac{\theta}{2}} \cdot \Big(\left(\sin(\beta+\gamma+\theta) - \sin(\beta+\gamma)\right)\Delta 1 + \left(\cos(\beta+\gamma+\theta) - \cos(\beta+\gamma)\right)\cdot\Delta 2 \\ \qquad - \left(\cos(\alpha+\gamma+\theta) - \cos(\alpha+\gamma)\right)\cdot L\theta \Big) \\ \Delta Y = \dfrac{1}{2\sin\frac{\theta}{2}} \Big(-\left(\cos(\beta+\gamma+\theta) - \cos(\beta+\gamma)\right)\cdot\Delta 1 + \left(\sin(\beta+\gamma+\theta) - \sin(\beta+\gamma)\right)\Delta 2 \\ \qquad - \left(\sin(\alpha+\gamma+\theta) - \sin(\alpha+\gamma)\right)\cdot L\theta \Big) \\ \Delta\theta = \theta \end{cases}$$

反算标定 L, α.

a

$$\Delta X = \frac{1}{2\sin\frac{\theta}{2}} \cdot \Big(\left(\sin(\beta+\gamma+\theta) - \sin(\beta+\gamma)\right)\Delta 1 + \left(\cos(\beta+\gamma+\theta) - \cos(\beta+\gamma)\right)\cdot\Delta 2 - \left(\cos(\alpha+\gamma+\theta) - \cos(\alpha+\gamma)\right)\cdot L\theta \Big)$$

$$\Delta y = \frac{1}{2\sin\frac{\theta}{2}}\Big(\overbrace{-(\cos(\beta+\gamma+\theta)-\cos(\beta+\gamma))\cdot\Delta 1 + (\sin(\beta+\gamma+\theta)-\sin(\beta+\gamma))\Delta 2}^{b}$$

$$-(\sin(\alpha+\gamma+\theta)-\sin(\alpha+\gamma))\cdot L\theta\Big)$$

则 $\Delta x = \frac{a}{2\sin\frac{\theta}{2}} - \frac{\theta}{2\sin\frac{\theta}{2}}\cdot(\cos(\alpha+\gamma+\theta)-\cos(\alpha+\gamma))\cdot L$

因 θ 很小，$\frac{\theta}{2\sin\frac{\theta}{2}} \simeq 1$

故 $$\begin{cases} \Delta x = \frac{a}{2\sin\frac{\theta}{2}} - (\cos(\alpha+\gamma+\theta)-\cos(\alpha+\gamma))\cdot L \\ \Delta y = \frac{b}{2\sin\frac{\theta}{2}} - (\sin(\alpha+\gamma+\theta)-\sin(\alpha+\gamma))\cdot L \end{cases}$$

累加得: $\sum_{i=0}^{n}\Delta x = \sum_{i=0}^{n}\frac{a}{2\sin\frac{\theta_i}{2}} - L\cdot\sum_{i=0}^{n}(\cos(\alpha+\gamma+\theta_i)-\cos(\alpha+\gamma))$

对于 $\sum_{i=0}^{n}(\cos(\alpha+\gamma+\theta_i)-\cos(\alpha+\gamma))$

$$= \cos(\alpha+\gamma_0+\theta_0)-\cos(\alpha+\gamma_0)+\cos(\alpha+\gamma_0+\theta_0+\theta_1)-\cos(\alpha+\gamma_0+\theta_0)$$

$$+ \cdots\cdots + \cos(\alpha+\gamma_0+\sum_{i=0}^{n}\theta_i)-\cos(\alpha+\gamma_0+\sum_{i=0}^{n-1}\theta_i)$$

$$= \cos(\alpha+\gamma_0+\sum_{i=0}^{n}\theta_i)-\cos(\alpha+\gamma_0) = \cos(\alpha+\gamma)-\cos(\alpha+\gamma_0)$$

标定时可取 $\gamma_0 = 0$

故 $L\cdot(\cos(\alpha+\gamma)-\cos\alpha) = \overbrace{\sum_{i=0}^{n}\frac{a}{2\sin\frac{\theta_i}{2}} - \sum_{i=0}^{n}\Delta x}^{k_1}$

同理: $L\cdot(\sin(\alpha+\gamma)-\sin\alpha) = \overbrace{\sum_{i=0}^{n}\frac{b}{2\sin\frac{\theta_i}{2}} - \sum_{i=0}^{n}\Delta y}^{k_2}$

故: $\frac{k_1}{k_2} = \frac{\cos(\alpha+\gamma)-\cos\alpha}{\sin(\alpha+\gamma)-\sin\alpha} \quad (\gamma \neq 2k\pi)$

可解得: $\alpha = \arctan\frac{k_2(\cos\gamma-1)-k_1\sin\gamma}{k_1(\cos\gamma-1)+k_2\sin\gamma}$

不妨令 $\gamma=\pi$ 则 $\alpha=\arctan\frac{k_2}{k_1}$

有 $k_1^2+k_2^2=L^2\cdot 2[1-\cos(\alpha+\gamma)\cos\alpha-\sin(\alpha+\gamma)\sin\alpha]$

$=2L^2\cdot(1-\cos\gamma)$

故 $L=\sqrt{\frac{k_1^2+k_2^2}{2(1-\cos\gamma)}}$ 令 $\gamma=\pi$, $L=\frac{\sqrt{k_1^2+k_2^2}}{2}$

k_1, k_2 可记录